Atoms to Universe
Physics 340
Assignment 4

1. Aristotle, Newton, and Einstein all had different theories of what caused bodies to fall to the earth, or kept the planets moving. Compare and contrast their various theories.

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Aristotle.– There is a natural home for any body depending on its composition– either at rest on the ground (water or earth) or at rest in the sky (fire and air). Bodies try to go to their natural place. The Heavier the body, the more it would like to return to earth and the faster it does so.

Newton: Each objects exerts and attractive force on any other body. That force is proportional to the mass of the body, and the mass of the other body, is directed along the line joining the centers (of mass) of the two bodies, and falls off inversely as the distance between the two centers and is directed toward the attracting body. The motion of the bodies obeys $\vec{F} = m\vec{a}$ where $\vec{F}$ is the force (both in its direction and its magnitude), $m$ is the mass of the body the force is applied to , and $\vec{a}$ is the acceleration of the body the force is applied to .

Einstein: The usual force of gravity does not exist, and bodies acted upon purely by gravity follow straight lines in spacetime (Newton's first law). However the structure of the distances in spacetime are such that those straight lines actually cause al lof the effects usually associated with gravity. In particular, in the simplest case, gravity is the inequable flow of time from place to place. Those differenced distances in the time direction mean that the "shortest distance" straight line actually looks like it is what Newton would have predicted with his forces.

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2. In special relativity the quantity $\sqrt{1-\frac{v^2}{c^2}}$ (the square root of the (one minus the speed squared the object over the velocity of light squred) plays a crudial role. The length contraction is by that factor. The time dilation is by that factor.

Consider a traveller who is a twin who decides to take a trip of his own to Alpha Centaurus, the nearest star to the earth, about 4 light years away (How far away is it in kilometers?)

———————————————————- 4 light years = $4{\cdot}\pi 10^7 seconds/year\cdot 300000km/s = 38\cdot 10^12km$

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According to his twin on earth, he travels at .8 of the speed of light and does not stop on Alpha Centaui and turns around and flies back at the same speed. How long would it take for him to get to Alpha Centauri and back?

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He travels at .8 c a distance of 8 light years, wwhich would take 8/.8=10 years according to his brother on earth

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How much older would his brother on earth be?

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His brother on earth would be 10 years older.

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How much older would he be when he came back?

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His time would be $\sqrt{1-(.8c/c)^2}10yr = \sqrt{1-.64}10yr = 6yr$ older.

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3. Explain in some way the "paradox" of the pole in the garage. (Assume that at rest a pole and a garage are the same length. The pole now approaches the garage as .8 times the velocity of light. The pole appears shorter (How much?) from the viewpoint of the garage, and easily fits in. However according the person riding beside the pole the garage is shorter (by how much?) and there is no way the pole can fit. What is going on here?

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There are two aspects to this. One is the "length contraction" and the other is simuleneity. When one says " the pole is in the grage" if the pole or garage is moving, one means that at some fixed time, both the front and back end of the pole are inside the garage. There are two aspects to the problem for the two observers. One is that they do not share a concept of simulteneity. Ie, they do not agree what the concept of "the same time" means.

For the barn, the pole looks $\sqrt{1-(.8)^2} = .6$ times the length.It easily fits into the barn. for the pole, the barn looks .6 times thelength and there is no way it could fit in. The spacetime diagram also makes this clear. The barn's idea os simultaneity is horizontal lines, and it is clear the pole ( the two red lines indicating the front and back of the pole) fit in the barn. The pole for his is .6 times as long as the barn. On the other hand the blue line is the simulteneity for the pole, and it is clear that the pole does not fit in the barn if that is what it considers to be the "same time".

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4)Aberation: Consider someone looking at a pole sticking up from the ground. He suddenly begins to move at close to the speed of light toward the pole. what happens to the height of the pole as seen by the person just after he begins to move? Recall what Bradley found for Gamma Draconis. (You do not need to solve this numerically)

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In the transverse direction to the motion, both frames agree on the length of an object. The light coming from the top and bottom of the pole have a certain angle depending on how far away the observer is. The observer now moves toward the pole. Like the angle that rain seems to fall, the direction the light from the top and bottom seems to be coming from is now closer to the horizontal. Ie, the angle that the top and bottom of the pole are seen as is smaller than when the observer was not moving, at least just as he starts moving. The pole seems to be further away. If it is further away, according to the observer, it must have taken light longer to get to him. Thus to that

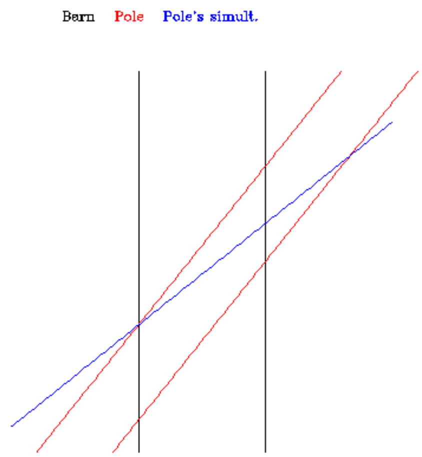


Figure 1: the red lines indicate the pole's trajectory in the barn's (black lines) frame and the blue line indicates the poles idea of what is simultaneous.

observer, that light must have left further in the past than it did just before he started moving. If he plots the position and time of the pole, it will seem not only to be further away, but also further back in time, and will seem to have gone into the past of where it was just before he started moving.

See the paper
W.G. Unruh –Parallax distance, time, and the twin "paradox" *American Journal of Physics* **49**, 589 (1981); https://doi.org/10.1119/1.12464
for an "explanation" of the twin's paradox using this way of defining distance.

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5)a)Consider a disk rotating so that its speed at the circumference is .6 times the speed of light. Assume that the radius of the disk is 1 meter. What happens to the cicumference according to special relativity? What happens the radius of the disk?

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The circumference is in the direction in which the disk is moving, so to a stationary observer, the measuring sticks of the person on the disk will appear to be shorter. This means that more of them will fit around the circumference of the disk. The radius is perpendicular to the direction of motion, so it will be the same length– the same number of meter sticks will fit around from the center to the edge. Thus the person on the disk will say that the circumference of the disk will be larger than $2\pi r$, as if it were curved. Thus the disk will seem to be seriously warped to the rotating observer.

This was an argument that Einstein made in order to argue that the geometry in an accelerating system (Huygens had already shown in 1660 that the motion in a circle is an accelerated motion).

This argument has a long history. See for example https://arxiv.org/abs/gr-qc/0207104

There is a problem with this. See the next part of the question. stationary observer.

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b)What would happen if the person riding on the circumference tried to synchronize the clocks around the circumference? (again you need only answer this qualitatively).

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To the stationary observer, the clocks will seems to be going more slowly. But simulteneity for the moving observer will be such that to the stationary observer, what the moving observer regards as simultaneous will be to the future in front and to the past behind the moving observer as far as the sationary observer is concerned. But of course as we go around the rim, being in the future ahead will eventually come around the same point eventually. Thus going around the rim, the moving observer will see himself out of sync with himself. Ie, there is no way that the moving observer can define simulteneity for all of the clocks around the rim consistantly. Note that this is a problem on the surface of the earth, for the purpose of synchronizing the clocks all around the earth. The earth travels at 1600km/hr= .00000148 times the speed of light. Around the earth (1/7 light second distance) this would make a difference in time of .00000021 sec. (ie, .21micro seconds) But clocks nowadays are far more accurate that that. Thus, if one exhanged radio signals between adjacent clocks on the earth, one would find as one went around the earth, that your own clock was out by 21 microseconds with itself. The actual synchronization of clocks is done, against relativity, as if the surface of the earth were stationary. Thus, as timed against these synchronized clocks, the travel time of light one way around the earth would be longer by 42microseconds that going the other way, violating the constancy of the velocity of light from the viewpoint of an observer moving with the surface of the earth.

This is relevant to the first part of the question. The shortening of lengths as seen in the moving frame comes from two parts, and one is that it depends on the synchronization of clocks in the moving frame. Trying to synchronise the clocks in the rotating frame leads to problems because one cannot complete the synchronization around the whole rim. One finds that the clock of the observer, in order to be synchronized with itself around the whole rim, has to be synchronized with a different time for itself. Ie, it is impossible for the rotating observer to define a consistant synchronization of his clocks. This also means that it is impossible to define a consistant length around the rim, since that depends on the synchronization of the clocks.

[Note: this problem turned out to be far more subtle than I expected when I set it. I am marking it by seeing what you make of it, and if you give some

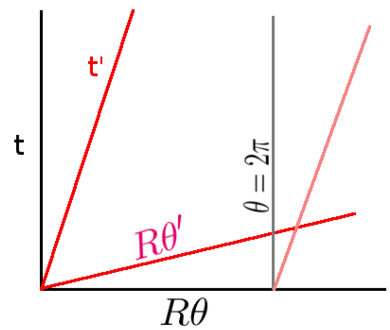


Figure 2: Here is the spacetime diagram of the edge, at radius R, of the rotating disk. The vertical is time as usual. The horizontal is the distance along the edge of the disk. The vertical red line at the left is the fixed point of a particular position of the edge of the rotating disk which is at the same point as $\theta = 0$ at time $t = 0$. It is moving at some velocity which is why the red line is slanted (it is at a different location $\theta$ at different times because of its motion). The more of less horizontal red line is the synchronization line for the clocks of the moving observer on the edge of the disk. The synchronization must be done a little distance at a time. –Eg, the person at $theta' = 0$ must send out a light signal to the person at $\theta' = \Delta\theta'$ where $\Delta\theta'$ is a small angle. Since to the moving observer, the velocity of light is $c$ he knows his clock should read $R\Delta\theta'/c$ when he receives the light. Having synchronized his clock, he can then send a signal to his neighbor, etc. The problem is that once on has gone around to $\theta = 2\pi$, a full circle as far as the non-roatating observer is concerned, we are back to the original position, and the light red line on the right should be the same line as the dark red line on the left. Ie, along the light red line the time should be the same (for the non-rotating observer) as along the dark red line. But not t'=0 should be at two different times. Since it is the identical clock to the one on the left, it should be synchronized in the same way as the one on the left, ie, where it intersects the horizontal black line. But according the light synchronization, it should also be zero were the horizonatl red line intersects the vertical pale-red line. And these two synchronizations are different. Similarly the length of the horizontal red line is longer that $R2\pi$, so the amount of angle around the rim $\theta'$ is larger than $2\pi$. From these measurements the person on the rotating rim will argue that the circle is warped. The angle around the circle is not equal to $2\pi$ (or 360 degrees). Einstein used this as an argument that special relativity required changes in the geometry of space to describe gravity.

arguments about the behaviour of an observer on the rotating disk with respect to the fixed observer. ]

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Note that the web page https://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/Reciprocity/index.html might help to understand special relativity paradoxes from a different point of view. The whole book, http://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/ might be helpful.

[ Brief table of commonly used prefixes: n = nano = $10^{-9}$ = 1/1,000,000,000
$\mu$ = micro = $10^{-6}$ = 1/1,000,000
m = milli = $10^{-3}$ = 1/1,000
c = centi = $10^{-2}$ = 1/100
d = deci = $10^{-1}$ = 1/10
h = hecta= $10^{2}$ = 100
K = kilo = $10^{3}$ = 1000
M = Mega = $10^{6}$ = 1,000,000
G = giga = $10^{9}$ = 1,000,000,000 ]

It is interesting that in scientific notation, names are given only up to Y= Yotta= $10^{24}$, whereas in classical Japanese there are names for numbers at least all the way up to $10^{52}$.

http://en.wikipedia.org/wiki/Japanese_numerals.

(The Japanese use 10000=$10^4$ as the multiple for names, rather than our 1000.) Why in the 16th century anyone would need to give such a large number a name I do not know. This aside is of course totally irrelevant to the course.