odes

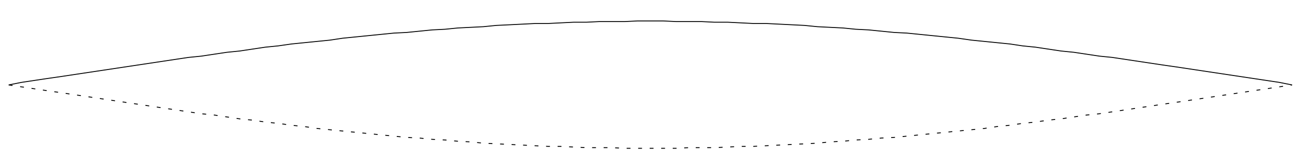
I. MODES OF VIBRATION

Any complex body (ie more complicated than a single mass on a simple spring) can vibrate in many different ways. Ie, there is no one "simple harmonic oscillator". These different ways of vibrating will each have their own frequency, that frequency determined by moving mass in that mode, and the restoring force which tries to return that specific distortion of the body back to its equilibrium position.

It can be somewhat difficult to determine the shape of these modes. For example one cannot simply strike the object or displace it from equilibrium, since not only the one mode liable to be exicted in this way. Many modes will tend to excited, and all to vibrate together. The shape of the vibration will thus be very complicated and will change from one instant to the next.

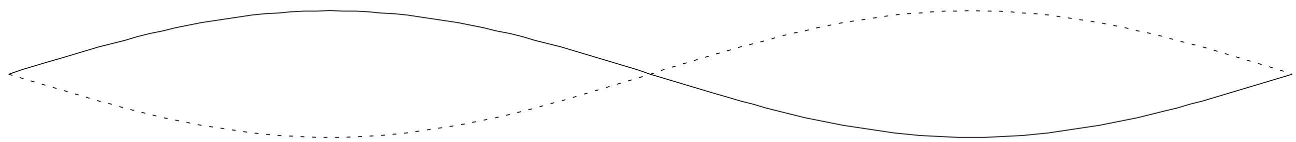
However, one can use resonance to discover both the frequency and shape of the mode. If the mode has a relatively high Q and if the frequencies of the modes are different from each other, then we know that if we jiggle the body very near the resonant frequency of one of the modes, that mode will respond a lot. The other modes, with different resonant frequencies will not respond very much. Thus the resonant motion of the body at the resonant frequency of one of the modes will be dominated by that single mode.

Doing this with strings under tension, we find that the string has a variety of modes of vibration with different frequencies. The lowest frequency is a mode where the whole string just oscillates back and forth as one– with the greatest motion in the center of the string.



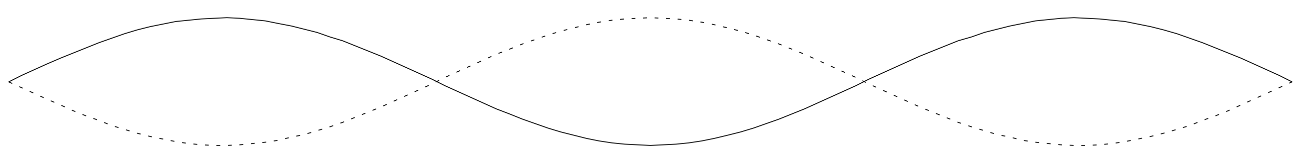
The diagram gives the shape of the mode at its point of maximum vibration in one direction and the dotted line is its maximum vibration in the other direction.

If we increase the frequency of the jiggling to twice that first modes frequency we get the string again vibrating back and forth, but with a very different shape. This time, the two halves of the string vibrate in opposition to each other. As one half vibrates up, the other moves down, and vice vesa.



Again the diagram gives teh shape of this mode, with the solid line being the maximum displacement of the string at one instant of time, and the dotted being the displacement at a later instant (180 degrees phase shifted in teh motion from the first instant)

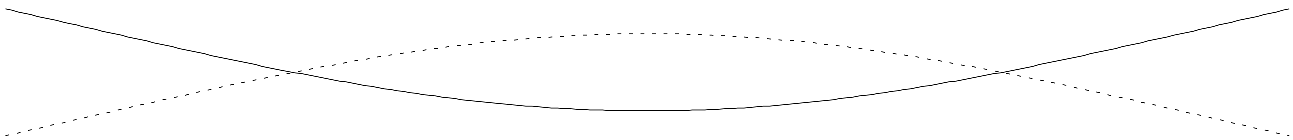
If we go up to triple the frequency of the first mode, we again see the string vibrating a large amount– ie at the resonant frequency of the so called third mode. In this case the string is divided into three equal length section, each vibrating in oposition to the adjacent piece.



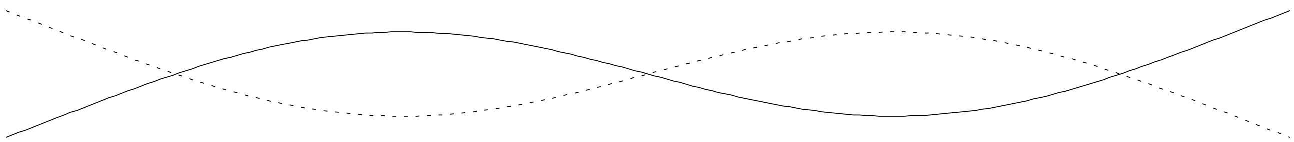
As we keep increasing the jiggling frequency we find at each whole number multiple of the first modes frequency another mode. At each step up, the mode gets an extra "hump" and also an extra place where the string does not move at all. Those places where the string does not move are called the **nodes** of the mode. Nodes are where the the quantity (in this case the displacement) of a specific mode does not change as the mode vibrates.

Modes of a bar

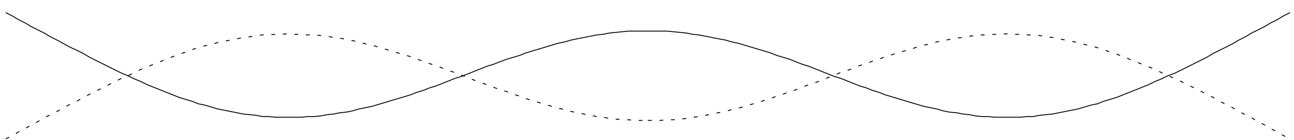
The modes of the string have the special feature that the frequencies of all of modes are simply integer multiples of each other. The n^{th} mode has a frequency of n times times the frequency of the first mode. This is not a general feature of modes. In general the frequencies of the modes have no simple relation to each other. As an example let us look at the modes of a vibrating bar free bar. In the figure below, we plot the shape of the first five modes of a vibrating bar, together with teh frequencies of the five modes. Again the solid lines are the shape of the mode on maximum displacement in one direction and the dotted the shape on maximum displacement in the other direction. Note that these are modes where the bar is simply vibrating, and not twisting. If one thinks about the bar being able to twist as well, there are extra modes. For a thin bar, the frequencies of these modes tend to be much higher than these lowest modes discussed here. However the wider the bar, the lower the frequencies of these modes with respect to the vibrational modes.



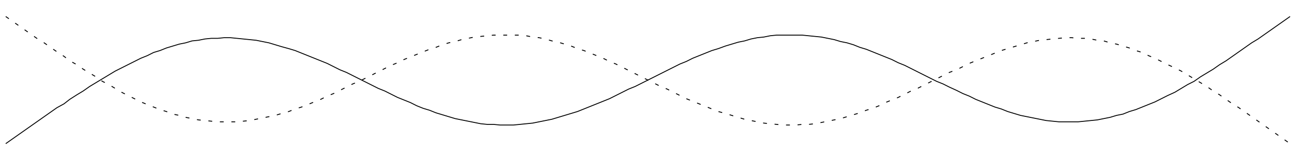
Mode 1 freq=f



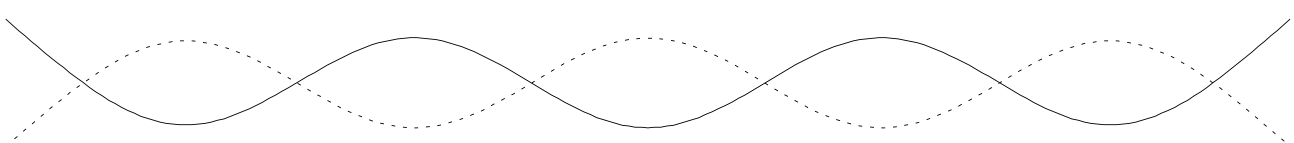
Mode 2 freq=2.75 f



Mode 3 freq=5.4 f



Mode 4 freq=8.9 f



Mode 5 freq= 13.3 f

Damping

We note that if we lightly hold a finger or other soft item against the vibrating object, it will vibrate against the finger unless the finger happens to be placed at a node where the bar does not vibrate at that node. We can see that the lowest mode and the fifth mode both have nodes at a point approximately 1/4 of the way along the bar. Thus if one holds the bar at that point and strikes the bar, then all of the modes will be rapidly damped except the first and fifth modes, which have a node there. Similarly, if one holds the bar in its center, the second, fourth modes both have nodes there while the others do not. Thus only those two will not be damped out.

We note that these modes do not have any nice relation between the frequencies of their modes. We note also that if we strike the bar, we can hear a number of different pitches given off by the bar. For example if we hold it at the 1/4 point, we hear two frequencies, one a very low one and another very high (13.3 times the lowest).

On the other hand if we strike or pluck a string, we hear only one pitch, even if we do not damp out any of the modes. Is there something strange about how the string vibrates? The answer is no. The string vibrates with all of its modes, just as the bar does. It is our mind that is combining all of the frequencies of the various modes into one pitch experience.